Limits & Functions Test

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1. Domain of f(x) = $\sqrt{x^2 - 9}$ is:

a) $|x| \geq 3$ b) $|x| < 3$ c) $|x| > 3$ d) $|x| \leq 3$

$f(x) = \sqrt{x^2 - 9}$

Domain $\Rightarrow |x| \geq a$

$f(x) = \sqrt{x^2 - 3^2}$

Domain $\Rightarrow |x| \geq 3$

Range $\Rightarrow [0, \infty]$

2. Which of the following is explicit function?

a) $x^2 + xy + y^2 = 7$

b) $\frac{xy^2 - y + 1}{xy} = 9$

c) $x^2 + y + 2x - 1 = 0$

d) $x^2 - xy + y^2 = 0$

$x^2 + y + 2x - 1 = 0$

$y = -x^2 - 2x + 1 = 0$

3. Which of the following is not true?

a) $\sin h^2 x + \cos h^2 x = 1$

b) $1 + \tan h^2 x = sech^2 x$

c) $\cos h^2 x - \sin h^2 x = \text{sech}2x$

d) All of these

4. If g(x) = $\frac{2}{x}$, then $g^2(x)$ = ?

a) $\frac{4}{x^2}$ b) $\frac{2}{x}$ c) x d) $\frac{4}{x}$

$g^2(x)$ = gog(x)

$g^2(x)$ = g(g(x))

$g^2(x)$ = g($\frac{2}{x}$)

$g^2(x) = \frac{2}{\frac{2}{x}}$

$g^2(x) = \frac{1}{\frac{1}{x}}$

$g^2(x)$ = x

5. Given that gf(x) = $4x^2$ + 4x if f(x) = 2x + 1then g(x) =

a) $4x^2$ b) x^2 - 1 ⟵ c) x^2 + 1 d) x + x^2

gf(x) = $4x^2$ + 4x

g(2x+1) = $4x^2$ + 4x

f(x) = 2x + 1

g(x) = x^2 - 1

g(2x+1) = $(2x+1)^2$ - 1

g(2x+1) = $4x^2$ + 1 + 4x - 1

6. $\lim_{x-0}(\frac{4x+3}{x-3})$ = ?

a) -1 b) 0 c) 2 d) 1

$$\lim_{x-0}(\frac{4x+3}{x-3})$$

$$\frac{3}{-3} = -1$$

7. $\lim_{x-4}(\frac{x-4}{\sqrt{x}-\sqrt{4}})$ = ?

a) $2\sqrt{2}$ b) 0 c) $\frac{1}{4}$ d) 4

$\lim_{x-4}(\frac{x-4}{\sqrt{x}-\sqrt{4}})$

$\lim_{x-4}(\frac{1}{\frac{1}{2\sqrt{x}}})$

$\frac{1}{\frac{1}{2\sqrt{4}}}$

$\frac{1}{\frac{1}{2(2)}}$

$\frac{1}{\frac{1}{4}} = 4$

8. $\lim_{t-\infty}(\frac{t}{t+1})^t =$

a) e b) e^{-2} c) e^{-1} d) e^2

$$\lim_{t-\infty}(\frac{t}{t+1})^t$$

$$e^{\frac{1}{t}\times -t}$$

$$\lim_{t-\infty}(\frac{t+1}{t})^{-t}$$

$$e^{-1}$$

$$\lim_{t-\infty}(\frac{t}{t}+\frac{1}{t})^{-t}$$

$$\lim_{t-\infty}(1+\frac{1}{t})^{-t}$$

9. For what value of m $\lim_{x-(-1)} f(x)$ exists, if

f(x) = $x + 4 , x \leq -1$

$m + 2 , x \geq -1$

a) m = 2 b) m = -1 c) m = 1 d) m = -4

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x)$

-1 + 4 = m + 2

3 = m + 2

m = 1

10. The graph of $x^2 + y^2 = 9$ is symmetric about:

a) x-axis b) origin c) y-axis d) All of these

$x^2 + y^2 = 9$

x → -x Symmetric about y-axis

y → -y Symmetric about x-axis

x → -x ; y → -y Symmetric about origin

11. The function $f(x) = \frac{x^2-1}{x-1}$ has a vertical asymptote at;

a) x = 0 b) x = 1 c) x = -1 d) $x = \frac{1}{2}$

$f(x) = \frac{x^2-1}{x-1}$

For vertical asymptote check where function becomes undefined.

12. If f(x) = 3x + 7 then $f^{-1}(2)$ = ?

a) 3 b) $\frac{-5}{3}$ c) $\frac{6}{7}$ d) 13

f(x) = 3x + 7

$f^{-1}(x)$ = ?

f(x) = 3x + 7

$x = 3f^{-1}(x) + 7$

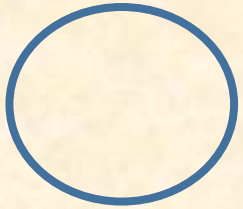
$f^{-1}(x) = \frac{x-7}{3}$

$f^{-1}(2) = \frac{2-7}{3}$

$f^{-1}(2) = \frac{-5}{3}$

13. The graph of $y = -\sqrt{a^2 - x^2}$ represents:

a) Ellipse b) Parabola c) Hyperbola d) Semi-circle

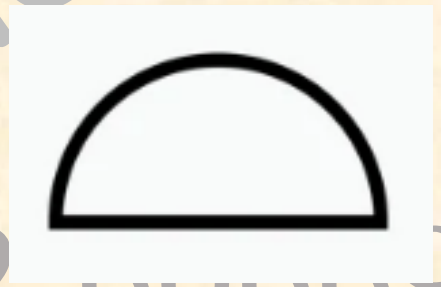


$x^2 + y^2 = a^2$

$y^2 = a^2 - x^2$

$y = \pm\sqrt{a^2 - x^2}$

$y = -\sqrt{a^2 - x^2}$, $y = +\sqrt{a^2 - x^2}$



14. $f(x) = ax + b$ and $g(x) = cx + d$ then $f(g(x)) = g(f(x))$ only if:

a) $f(a) = g(a)$ b) $f(b) = g(a)$ c) $f(d) = g(b)$ d) $f(c) = g(a)$

$f(g(x)) = g(f(x))$

$a(cx+d) + b = c(ax+b) + d$

$acx + ad + b = acx + cb + d$

$a(d) + b = c(b) + d$

$f(d) = g(b)$

15. If the function f, g and h are defined by $f(x) = x^3$, $g(x) = x - 2$ and $h(x) = \frac{x}{2}$ then $f(g(h(8))) = ?$

a) 0 b) 27 c) 4 d) 8

$f(x) = x^3$

$g(x) = x - 2 \implies g(4) = 4 - 2 = 2$

$h(x) = \frac{x}{2} \implies h(8) = \frac{8}{2} = 4$

$f(g(h(8))) = ?$

$f(g(4)) \implies f(2) = ?$

$f(2) = (2)^3 = 8$

16. $\lim_{x \to e} \frac{lnx-1}{x-e} = ?$

a) 1 b) $\frac{1}{2}$ c) $\frac{1}{e}$ d) 0

$$\lim_{x \to e} \frac{\frac{1}{x} - 0}{1 - 0}$$

$$= \frac{1}{e}$$

17. $coth^{-1}x$ = ?

a) $\frac{1}{2}\ln(\frac{x-1}{x+1})$

b) $\frac{1}{2}\ln(\frac{x+1}{x-1})$

c) $\ln(x+\sqrt{x^2-1})$

d) $\ln(\frac{1}{x}+\frac{\sqrt{1-x^2}}{x})$

18. Range of $y = 2 - \sqrt{x+2}$ is:

a) $(-\infty, 2]$ b) $[-\infty, 2)$ c) $(-\infty, 2)$ d) $[2, \infty)$

$y = 2 - \sqrt{x+2}$

$2 - [0, \infty)$

$(-\infty, 2]$

19. $f(x) = \frac{x^3-x}{x^2+1}$ is …….. Function?

a) Even b) odd c) Neither even nor odd d) None of these

$$f(x) = \frac{x^3-x}{x^2+1}$$

$$f(-x) = \frac{(-x)^3-(-x)}{(-x)^2+1}$$

$$f(-x) = \frac{-x^3+x}{x^2+1}$$

$$f(-x) = \frac{-(x^3-x)}{x^2+1}$$

$$f(-x) = -f(x)$$

20. f(x) = $\frac{2x+1}{x-1}$, f^{-1}(f(x)) = ?

a) x b) –x c) $\frac{x-1}{2x+1}$ d) None of these

21. f(x) = $\frac{2x-1}{3x+2}$ domain of $f^{-1}(x)$ is:

a) R – $\{\frac{3}{2}\}$ b) R – $\{\frac{2}{3}\}$ c) R – $\{\frac{-2}{3}\}$ d) R = $\{-\frac{4}{3}\}$

Domf^{-1} = Range f

f(x) = $\frac{2x-1}{3x+2}$

R – $\{\frac{2}{3}\}$

22. $\lim_{x\to 3} \frac{x^n - 3^n}{x-3} = 6$, n = ?

a) 1 b) 2 c) 3 d) 4

$$\lim_{x\to 3} \frac{x^n - 3^n}{x-3} = 6$$

$$\lim_{x\to 3} \frac{nx^{n-1} - 0}{1} = 6$$

$$n(3)^{n-1} = 6$$

$$(2)(3)^{2-1} = 6$$

$$6 = 6$$

23. $\lim_{x\to\infty} \frac{2x^2-3x-1}{3x^2-5x+1}$ = ?

a) ∞ b) 0 c) $\frac{3}{2}$ d) $\frac{2}{3}$

24. $\lim_{x\to\infty} \frac{sinx}{x}$ = ?

a) ∞ b) 1 c) 0 d) -1

$\lim_{x\to\infty} \frac{sinx}{x}$

y = sinx

Dom = R

Range = [-1, 1]

$\frac{1}{\infty}$ = 0

25. $\lim_{x\to 0}(1+\frac{1}{x})^{\frac{3x}{2}} = ?$

a) $e^{\frac{3}{2}}$ b) $e^{\frac{2}{3}}$ c) $e^{\frac{-3}{3}}$ d) None of these

$$\lim_{x\to 0}(1+\frac{1}{x})^{\frac{3x}{2}}$$

$$\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$$

26. $f(x) = \frac{x^2-1}{x-1}$ is …… at x = 1

a) continuous

b) Discontinuous

c) May be continuous or discontinuous

d) Impossible to determine

$$f(x) = \frac{x^2-1}{x-1}$$

27. $f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$, m = ? , n = ?

If f(x) is continuous at x = 3

a) m = -1 , n = 3

b) m = -1, n = -3

c) m = 1, n = 3

d) m = 1 , n = -3

$\lim_{x \to 3} f(x)$ exists

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$

$\lim_{x \to 3^-} (mx) = \lim_{x \to 3^+} (-2x + 9)$

3m = -2(3) + 9

3m = 3

m = 1

$\lim_{x \to 3} f(x)$ = f(3)

3 = n

28. Range of $y = \frac{1}{x-2}$ is:

a) R – {0} b) R – {2} c) R d) None of these

$y = \frac{1}{x-2}$

$\frac{a}{b} = 0$

R – {0}

29. $\lim_{x\to 0}(\frac{1-cosx}{sin^2x})$= ?

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{-1}{3}$ d) None of these

$$\lim_{x\to 0}(\frac{1-cosx}{sin^2x})$$

$$\lim_{x\to 0}(\frac{0-(-sinx)}{2sinxcosx})$$

$$\lim_{x\to 0}(\frac{sinx}{2sinxcosx})$$

$$\lim_{x\to 0}(\frac{1}{2cosx})$$

$$\frac{1}{2cos0}=\frac{1}{2}$$

30. Given that f(x) = 7 for all x then f(x + 3) = ?

a) 0 b) x + 3 c) 7 d) 10

f(x) = 7

f(x + 3) = 7