12 Math Chapter 02

Derivatives Work Sheet

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Given that the curve $y = ax^2 + \frac{b}{x}$ has a gradient 4 at the point (1, 5).

Calculate the values of a and b:

a) a = 3, b = 2
b) a = -3 , b = 2
c) a = 3, b = -2
d) a = -3 , b = -2

$$y = ax^2 + \frac{b}{x}$$

$$\frac{dy}{dx} = 2ax + b(-1)(x^{-2})$$

$$\frac{dy}{dx} = 2ax - \frac{b}{x^2}$$

$$\frac{dy}{dx})_{(1,5)} = 4$$

$$4 = 2a(1) - \frac{b}{(1)^2}$$

2a – b = 4

2(3) – (2) = 4

4 = 4

Only option a satisfied the above equation

If f(x) = $\frac{5}{x^2-1}$ then $f^{/}(x)$ =

a) $\frac{10x}{x^2-1}$ b) - $\frac{10x}{(x^2-1)^2}$ c) $\frac{10x}{(x^2-1)^{-2}}$ d) $\frac{-10x}{x^2-1}$

f(x) = $5(x^2-1)^{-1}$

f(x) = -5$(x^2-1)^{-2}$(2x)

f(x) = -10x$(x^2-1)^{-2}$

f(x) = $\frac{-10x}{(x^2-1)^2}$

$\frac{d}{dx}(x^n)$ = nx^{n-1} where:

a) n ϵ R　　b) n ϵ Z　　c) n ϵ Q　　d) n ϵ C

n ϵ Q

$$\frac{d}{dx}\frac{1}{\sqrt{lnx}} =$$

a) $\frac{lnx}{x}$

b) $\frac{(lnx)^3}{2x}$

c) $\frac{-1}{2x(lnx)^{3/2}}$

d) $\frac{x}{(lnx)^{3/2}}$

$$\frac{d}{dx}\frac{1}{\sqrt{lnx}} = \frac{d}{dx}(lnx)^{-1/2}$$

$$\frac{-1}{2}(lnx)^{-3/2}(\frac{1}{x})$$

$$\frac{-1}{2x(lnx)^{3/2}}$$

The last non-zero derivative of f(x) = $30x^7 + 5$ is:

a) 30!   b) $30 \times 7!$ c) 7!   d) $210x^6$

$$f(x) = 30.x^7 + 5$$

$$f^{/}(x) = 30.7x^6$$

$$f^{//}(x) = 30.7.6.x^5$$

Continuing in this way we get,

$$f^{vii}(x) = 30.7.6.5.4.3.2.1$$

$$f^{vii}(x) = 30 \times 7!$$

If y = lnx then $\frac{d^n y}{dx^n}$ =

a) $\frac{(-1)^n n!}{x^{n+1}}$ b) $\frac{(-1)^n}{x^{n-1}}$ c) $\frac{(-1)^{n+1}}{x^{n+1}}$ d) $\frac{(-1)^{n-1}(n-1)!}{x^n}$

y = lnx

$\frac{dy}{dx} = \frac{1}{x} = x^{-1}$

$\frac{d^2y}{dx^2}$ = (-1)x^{-2}

$\frac{d^3y}{dx^3} = (-1)^2 2x^{-3} = (-1)^2 2!\, x^{-3}$

On generalizing

$\frac{d^n y}{dx^n} = (-1)^{n-1}(n-1)!\, x^{-n}$

$\frac{d^n y}{dx^n} = \frac{(-1)^{n-1}(n-1)!}{x^n}$

$\frac{d^{35}}{dx^{35}}(\cos x) =$

a) Sinx b) –sinx c) cosx d) -cosx

After every fourth derivative function f(x) = cosx repeats, so divide 35 by 4 and get remainder 3. so we have to take the only third derivative. Which is sinx so answer is option (a).

$\frac{d}{dx}(\pi^x) =$

a) x.π^{x-1} b) π^x c) lnπ. π^x d) 0

π^x.lnπ(1)

If y = e^{-ax} then:

a) $y_n = a^n$y   b) y_n = -a^ny

c) $(-1)^n$y   d) $(-a)^n$y

y = e^{-ax}

y_1 = (-a) e^{-ax}

y_2 = (-$a)^2$ e^{-ax}

.

.

y_n = $(-a)^n$ e^{-ax}

y_n = $(-a)^n y$

If y = $\log_x e$, then $\frac{dy}{dx}$ is equal to:

a) $\frac{1}{x(\ln x)^2}$ b) $\frac{1}{x}$ c) $\frac{1}{x\ln x}$ d) $\frac{1}{e}$

$$y = \log_x e$$

$$= \frac{\ln e}{\ln x} = \frac{1}{\ln x} = (\ln x)^{-1}$$

$$y_1 = -(\ln x)^{-2}(\frac{1}{x}) = \frac{1}{x(\ln x)^2}$$

$\frac{d}{dx}$ | x – 1 | does not exist for:

a) x = 0 b) x = 1 c) x = -1 d) None of these

f(x) = | x – 1 |

Since $f^{/}(a) = lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

So, $f^{/}(1) = lim_{x\to 1}\frac{f(x)-f(1)}{x-1}$

$= lim_{x\to 1}\frac{|x-1|-0}{x-1} = lim_{x\to 1}\frac{|x-1|}{x-1}$

$\text{L}f^{/}(1) = lim_{x\to 1^{-}}\frac{|x-1|}{x-1}$

$\text{L}f^{/}(1) = lim_{x\to 1^{-}}\frac{-(x-1)}{x-1}$

$\text{L}f^{/}(1) = lim_{x\to 1^{-}}(-1)$ = -1

$\text{R}f^{/}(1) = lim_{x\to 1^{+}}\frac{|x-1|}{x-1}$

$\text{R}f^{/}(1) = lim_{x\to 1^{+}}\frac{(x-1)}{x-1} = 1$

$\text{L}f^{/}(1) \neq \text{R}f^{/}(1)$

So derivative at x= 1 does not exist

The minimum value of f(x) = 4cosx + 3 is:

a) -1 ⟸ b) -2 c) $-\sqrt{2}$ d) 1

$f(x) = 4\cos x + 3$

Range of cosx is [-1, 1]

So range of 4[-1, 1] + 3= [-1, 7]

Min. value of f(x) = -1

Max. value of f(x) = 7

$$\frac{d}{dx}(log_3x) =$$

a) $\frac{1}{x}$

b) $\frac{1}{xln3}$ ⬅

c) $\frac{1}{xlog3}$

d) $\frac{1}{log3}$

$$\frac{d}{dx}(log_ax) = \frac{1}{x} . \frac{1}{lna}$$

$$\frac{d}{dx}(log_3x) = \frac{1}{x} . \frac{1}{ln3} = \frac{1}{xln3}$$

Find derivative w.r.t x of $\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$:

a) $\frac{-2}{(x-1)^2}$ ⬅ b) $\frac{1-\frac{1}{x^2}}{(1-\frac{1}{x})^2}$ c) $\frac{(1+x)^2}{(1-x)^2}$ d) None of these

$$\frac{1+\frac{1}{x}}{1-\frac{1}{x}} = \frac{\frac{x+1}{x}}{\frac{x-1}{x}} = \frac{x+1}{x-1}$$

$= \frac{x+1}{x-1}$ Apply quotient rule on it and answer is option a.

Which of the following Mathematician gave the notation Df(x) for the derivative?

a) Newton b) Cauchy c) Lagrange d) Euler

Name of Mathematician	Leibniz	Newton	Lagrange	Cauchy
Notation used for derivative	$\frac{dy}{dx}$ or $\frac{df}{dx}$	f(x)	$f^{/}$(x)	Df(x)

For which of the following intervals is the function y = x^2 - 6x + 5 increasing?

a) 1 < x < 5 b) x > 5 c) x > 3 d) x < 1

A function is increasing when its first derivative is greater than 0.

y = x^2 - 6x + 5

$\frac{dy}{dx}$ = 2x - 6

2x – 6 > 0

x > 3

Ripples in shape of circles are produced when a stone is thrown in water. If the rate of change of circumference of these circles is 12π, what is the rate of change of Area of the circle when the radius is 3cm?

a) 28π b) 36π ⟵ c) 54π d) 60π

Rate of change of circumference of circle = $\frac{dC}{dt} = 12\pi$

Circumference = C = $2\pi r$

$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{2\pi} \frac{dC}{dt}$

$\frac{dr}{dt} = \frac{1}{2\pi}(12\pi) = 6$

$\frac{dr}{dt}$ = 6 = rate of change of radius

Area of circle = A = πr^2

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(3)(6) = 36\pi$

What is the derivative of $2t^3$ w.r.t $3t^2$?

a) $2t^3$ b) t ⟸ c) t^2 d) $\frac{1}{t}$

y = $2t^3$ and x = $3t^2$

$\frac{dy}{dt}$ = $6t^2$ and $\frac{dx}{dt}$ = 6t

We need to find the derivative of y = $2t^3$ w.r.t x = $3t^2$ ($\frac{dy}{dx}$)

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dy}{dx}$ = $6t^2 \times \frac{1}{6t}$ = t

What are the two numbers whose sum is 20 but their product is maximum?

a) 10, 10 b) 15, 5 c) 0, 20 d) 1, 19

Multiply the numbers in the given options and check which of them result in the maximum answer. It is obvious from the given options that 10, 10 will result in the maximum product.

Which of the following represents $\frac{dy}{dx}$ if sinx = e^y

a) –cotx b) tanx c) –tanx d) cotx

e^y = sinx

lne^y = lnsinx

ylne= lnsinx

y = lnsinx

$\frac{dy}{dx} = \frac{1}{sinx}(cosx)$

$\frac{dy}{dx}$ = cotx

If S = f(t) is a displacement function over time t, then the expression $\lim_{\Delta t \to 0} \frac{f^{/}(t+\Delta t)-f^{/}(t)}{\Delta t}$ is called:

a) Velocity

b) average velocity

c) Acceleration

d) Average acceleration

If s = f(t) then $f^{/}(t)$ represents the velocity and rate of change of velocity is called acceleration.

Find the speed of a moving particle in a straight line, whose position in meters after t seconds is given by s(t) = $t^2 + t$, after 3 seconds:

a) 12 m/s b) 6 m/s c) 7 m/s d) 5 m/s

s(t) = $t^2 + t$

$\frac{dS}{dt}$ = 2t + 1

v = 2(3) + 1

v = 7 m/s

If y = $e^{xlog_e a}$ then $y^/$ = ?

a) lnae^{xlna} b) e^x c) a^x d) $a^{xlog_e a}$

y = $e^{xlog_e a}$

y = e^{xlna}

$y^/ = e^{xlna}$.lna

If f(x) = sinx then $f^{/}(cos^{-1}x)$ = ?

a) cosx b) sinx c) -x d) x

f(x) = sinx

$f^{/}$(x) = cosx

$f^{/}(cos^{-1}x)$ = cos(cox^{-1}x)

$f^{/}(cos^{-1}x)$ = x

If y = $\theta^{tan\theta}$ then $\frac{dy}{d\theta}$ is:

$a)$ $\theta^{tan\theta}(\frac{1}{cos^2\theta} + \frac{tan\theta}{\theta})$

b) $\theta^{tan\theta}$(lnθ.$cos^2\theta$+$\frac{tan\theta}{\theta}$)

c) $\theta^{tan\theta}(\frac{ln\theta}{cos^2\theta} + \frac{tan\theta}{\theta})$

d) $\theta^{tan\theta}(\frac{ln\theta}{cos^2\theta} + \frac{\theta}{tan\theta})$

y = $\theta^{tan\theta}$

Take logarithm on both sides,

lny = $ln\theta^{tan\theta}$

lny = tanθ.lnθ

Now use product rule of derivative answer is option c.