12 Math Chapter 01

Functions and Limits

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1. If (2,5) lie in the graph of an odd function then which one must lie must on the graph of function:

a) (-2 , -5)

b) (-2 , 5)

c) (2 , -5)

d) (5 , 2)

An odd function satisfy the condition f (x , y) = f (-x , -y)

So it (2 , 5) is on the graph of f then (-2, -5) is also on the graph of f.

2. Domain of function y = $\frac{2x}{\sqrt{x^2-4}}$ is:

a) R – $\{\pm 2\}$

b) $|x| > 2, x \neq 0$

c) $|x| > 2$

d) R – $\{\pm 2,0)$

Function y = $\frac{2x}{\sqrt{x^2-4}}$ will be real and defined if $x^2 - 4 > 0 \Rightarrow x^2 > 4 \Rightarrow |x| > 2$

3. Which of the following is incorrect in general :

a) f^2 = fof

b) $fof^{-1} = 1$

c) gof = fog

d) All are incorrect

Commutative law is invalid in composition of functions.

4. $\lim_{x \to 0^-} \frac{\sin x}{\sqrt{x}}$ equal to:

a) 0 b) 1 c) $-\frac{1}{2}$ **d) does not exist**

Function is not defined on the left side of x = 0 (i.e for x < 0) so the left hand limit does not exist.

5. The equation of horizontal asymptote of function $y = \frac{4x+1}{2x-5}$:

a) $y = 2$ b) $x = \frac{5}{2}$ c) $x = \frac{-1}{4}$ d) $y = \frac{-1}{5}$

The equation of horizontal asymptote is $y = \frac{4}{2} = 2$

$\Rightarrow y = 2$

6. The set of point of discontinuity of the function f (x) = $\log_a$ x is:

a) ϕ b) (-∞, 0] c) (-∞, 0] d) (0,∞)

Set of point for which function is not defined is also a set of point of discontinuity . $\log_a$ x is not defined for non-positive numbers so required set is the interval (-∞. 0]

7. If $f(x) = (x + 1)^2$ and $g(x) = x - 1$, find $f(g(4))$

a) 4 b) 16 c) 25 d) 9

$f(g(x)) = f(x - 1) = ((x - 1) + 1)^2 = (x - 1 + 1)^2 = x^2$

$f(g(4)) = (4)^2 = 16$

8. A function is said to be even when:

a) f(x) = f(x)

b) f(-x) = f(x)

c) f(-x) = -f(x)

d) f(-x)f(-x)

f(-x) = f(x)

9. If f(x) = 2x +3, g(x) = x^3 , find gof

a) $2x^3$ +3 b) $(2x+3)^3$ c) $2x^3$ d) None of these

g (x) = x^3

gof (x) = g (f(x)) = g(2x + 3) = $(2x + 3)^3$

10. If f(x) = 2, find the value of f (x+2)

a) 4 b) 8 c) 2 d) 0

The function is defined as f(x) =2

This is a constant function which means for all value of x the value of f(x) will always be 2. i.e, even if the value of x is changed by adding 2, the value of f (x) will still be 2.

The graph of f(x) will be a horizontal line passing through the point (0,2). Hence c is the right answer.

11. Evaluate : $\lim_{x \to 0} \frac{\sin(6x)}{x}$

a) 0 b) 6 c) 1/6 d) does not exist

$$\lim_{x \to 0} \frac{\sin(6x)}{x} = \lim_{x \to 0} \frac{6 \sin(6x)}{6x} = 6 \lim_{x \to 0} \frac{\sin(6x)}{6x} = 6\,(1) = 6$$

12. If f (x) = x^2 + 1 , g (x) = 5x + 1, find f (g(x))

a) $25x^2 + 10x + 2$

b) $5x^2$ +6

c) $5x^3 + x^2 + 5x + 1$

d) none of these

f(g(x) denotes the composition of both functions . To find f(g)(x) we have to replace every occurrence of x in f (x) with the value g(x). So,

f(g(x)) = f (5x + 1)

= (5x + 1)^2 + 1 = $25x^2$ + 10x + 1 + 1 = $25x^2$ + 10x + 2

13. Vertical asymptotes of the graph f(x) = $\frac{x-2}{x^2-4}$ occur at:

a) x = -2 b) x = b c) y = 2 d) does not exist

Function is undefined at x = -2

14. If f(x) = $\frac{2}{\sqrt{x}}$, x > 0 find $f^{-1}(3)$ = ?

a) $\frac{2}{\sqrt{3}}$ b) $2\sqrt{3}$ c) $\frac{4}{9}$ d) $\frac{2}{3}$

f(x) = $\frac{2}{\sqrt{x}}$

y = $\frac{2}{\sqrt{x}}$

x = $\frac{2}{\sqrt{y}}$

$\sqrt{y} = \frac{2}{x}$

y = $\frac{4}{x^2}$

y = $\frac{4}{(3)^2}$

y = $\frac{4}{9}$

15. If f(x) = 2x + 3 and fffff(x) = ax + b, then:

a) a = 32 b) a = 64 c) a = 16 d) a = 8

fffff(x) = ax + b

$f^n(x) = a^n x + b$

$a^n = 2^5$

$a^n = 2^5 = 32$

16. $\lim_{x\to 0} \frac{e^{\frac{-x}{2}}}{1+e^{\frac{-x}{2}}} = ?$

a) 1 b) ½ c) 0 d) ∞

$$\lim_{x\to 0} \frac{e^{\frac{-x}{2}}}{1+e^{\frac{-x}{2}}}$$

$$\frac{e^{o}}{1+e^{0}}$$

$$\frac{1}{1+1} = \frac{1}{2}$$

17. $\lim_{n\to\infty}(\frac{5n+1}{5n})^n = ?$

a) $e^{1/5}$ b) e^5 c) e^{-5} d) $e^{-1/5}$

$$\lim_{n\to\infty}(\frac{5n+1}{5n})^n$$

After dividing numerator and denomirator from 5n we get,

$$\lim_{n\to\infty}(1+\frac{1}{5n})^n$$

$\lim_{n\to\infty}(1+\frac{1}{n})^n$ = e

$$(\lim_{n\to\infty}\left(1+\frac{1}{5n}\right)^{5n})^{1/5}$$

$(e)^{1/5}$

18. $\sum_{n=-\infty}^{n=\infty}(0.5)^n = ?$

a) Infinity b) 2 c) 1 d) 0

$$\sum_{n=-\infty}^{n=\infty}(\frac{1}{2})^n$$

$(\frac{1}{2})^{\infty} = \infty$

$(\frac{1}{2})^{-\infty} = 2^{\infty} = \infty$

When we add ∞ we get ∞

19. What is the value of $\lim_{x\to 0} \frac{\cos\left(\frac{3\pi}{2} - x\right) - \cos\left(\frac{3\pi}{2}\right)}{x}$ = ?

a) $\sqrt{2}$ b) -1 c) 1 d) limit does not exist

Apply “L'Hôpital's rule”

Take separate derivative of numerator and denominator then apply he limit.

Answer is -1

20. What is the value of $\lim_{h\to 0} \frac{|h|}{h}$ = ?

a) ± 1 b) 1 c) 0 d) limit does not exist

$|h| = \pm h$

So LHL ≠ RHL

Therefore limit does not exist

$\frac{e^{2x}-1}{2e^{x}}$ = ?

a) Sinx b) cosx c) sinhx d) coshx

$$\text{sinhx} = \frac{e^{x}-e^{-x}}{2} = \frac{e^{x}-\frac{1}{e^{x}}}{2} = \frac{e^{2x}-1}{2e^{x}}$$

22. Range of f(x) = $x^2 + 1$ is:

a) R b) R – {1} c) R – {-1} d) [1, ∞)

Range of x^2 is [0, ∞)

By adding 1 we get range of $x^2 + 1$ that is [1, ∞)

23. f(x) = $\frac{3x}{x^2+1}$ is ……function?

a) Even

b) Odd

c) Both Even and Odd

d) Neither even nor odd

f(x) = $\frac{3x}{x^2+1}$

f(-x) = $\frac{3(-x)}{(-x)^2+1}$

f(-x) = - $\frac{3x}{x^2+1}$

f(-x) = -f(x)

It is an odd function

24. What is the value of $\lim_{x\to 2} \frac{|x-2|}{x-2}$ = ?

a) 1 b) -1 c) 0 d) limit does not exist

$$\text{L.H.L} = \lim_{x\to 2} \frac{|x-2|}{x-2} = \lim_{x\to 2} -\frac{x-2}{x-2} = -1$$

$$\text{R.H.L} = \lim_{x\to 2} \frac{|x-2|}{x-2} = \lim_{x\to 2} \frac{x-2}{x-2} = 1$$

$$\text{L.H.L} \neq \text{R.H.L}$$

25. f(x) = secx tanx is a:

a) Odd function

b) Even function

c) Neither even nor odd

d) None of these

f(x) = secx tanx

f(-x) = sec(-x) tan(-x)

f(-x) = - secx tanx

f(-x) = - f(x)

26. The domain of $y = \sqrt{-x}$ is:

a) $(0, \infty)$ b) R c) $(-\infty, 0)$ d) $(-\infty, 0]$

To make $\sqrt{-x}$ positive we have to put negative values in place of x so domain is $(-\infty, 0]$.

27. The domain of the function $y = \frac{1}{\sqrt{16-x^2}}$ is:

a) (-4, 4) b) [-4, 4] c) R – (-4, 4) d) (4, ∞)

The domain of function is (-4, 4) because 4 and -4 we cannot put into the function otherwise function becomes undefined.

28. The range of the function $f(x) = \frac{1+x^2}{x^2}$ is:

a) [0, 1] b) (0, 1) c) (1, ∞) d) [1, ∞)

$f(x) = \frac{1+x^2}{x^2}$

$f(x) = 1 + \frac{1}{x^2}$

Range of $\frac{1}{x^2}$ is [1, ∞)

The range of $1 + \frac{1}{x^2}$ is:

The range of $1 + \frac{1}{x^2}$ is:

1 + [1, ∞) = [2, ∞)

So from given options c is correct

29. Which of the following is an implicit function?

a) y = x – 1

b) y – 1 = x^2

c) x^2 + y + 2 = 0

d) x^2+xy+y^2 = 9

x^2+xy+y^2 = 9 This is implicit function because y cannot be expressed easily in terms of x.

30. The only function which is both even and odd.

a) f (x) =a

b) f (x) = 0

c) F (x) = x

d) (a) & (*b*)

f(x) = 0 is both even as well as odd function.